

**INTERNATIONAL MATHEMATICS AND SCIENCE OLYMPIAD  
FOR PRIMARY SCHOOLS (IMSO) 2006  
Mathematics Contest in Taiwan, Exploration Problems**

Name: \_\_\_\_\_ School: \_\_\_\_\_ Grade: \_\_\_\_\_ ID number: \_\_\_\_\_

Answer the following 5 questions, and show your detailed solution in the answer sheet. Write down the question number in each paper. Each question is worth 8 points. Time limit: 60 minutes.

1. The solution to each clue of this crossnumber is a two-digit number. None of these numbers begins with zero. Complete the crossnumber, stating the order in which you solved the clues and explaining why there is only one solution.

**Clues Across**

- 1. A square number
- 3. A multiple of 11

**Clues Down**

- 1. A multiple of 7
- 2. A cube number

1.	2.
3.	

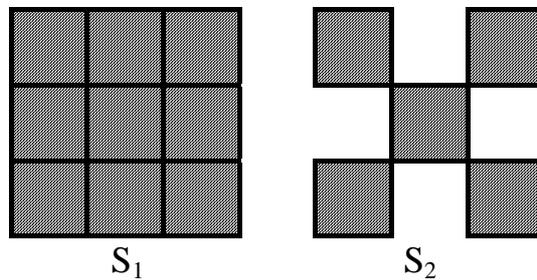
2. Notice that  $2^2 + 2^2 = 2^3$ , so two squares can sum to give a cube; however, the two squares here are equal (to 4).
- (a) Find two unequal squares whose sum is a cube.
  - (b) Show that there are infinitely many pairs of unequal squares whose sum is equal to a cube.
3. Is it possible to find a number  $11\dots 11$  that is divisible by 19?

4. The menu in the school cafeteria never changes. It consists of 10 different dishes. Peter decides to make his school lunch different everyday (at least 1 dish). For each lunch, he may eat any number of dishes, but no two are identical.
- (a) What is the maximum numbers of days Peter can do so?
- (b) What is the total number of dishes Peter has consumed during this period?

5. A sequence of shapes is made as follows.

- (1) Shape  $S_1$  is a shaded square of side 1 unit.
- (2) Shape  $S_2$  is made by dividing  $S_1$  into 9 equal squares and removing four of these, so that only the central and corner squares remain.
- (3) Shape  $S_3$  is made by applying the process in (2) to each of the squares of  $S_2$ .
- (4) Shape  $S_4$  is made by applying the process in (2) to each of the squares of  $S_3$ .

And so on.



- (a) Find the area and perimeter of shape  $S_3$ , giving your answers as fractions.
- (b) Find the least value of  $k$  for which the shape  $S_k$  has the area less than  $\frac{1}{30}$  and also has perimeter greater than 30.