World Youth Mathematics Intercity Competition

Team Contest Time limit: 60 minutes 2008/10/28

Chiang Mai, Thailand

Team: _____ ***Score:*** _____

1. The fraction $\frac{p}{q}$ is in the lowest form. Its decimal expansion has the form $0.abababab\ldots$ The digits a and b may be equal, except that not both can be 0. Determine the number of different values of p.

ANSWER : _____

World Youth Mathematics Intercity Competition

Team Contest Time limit: 60 minutes 2008/10/28

Chiang Mai, Thailand

Team: _____ ***Score:*** _____

2. Cover up as few of the 64 squares in the following 8×8 table as possible so that neither two uncovered numbers in the same row nor in the same column are the same. Two squares sharing a common side cannot both be covered.

6	4	5	7	7	3	3	5
4	8	4	3	6	7	5	1
3	1	5	7	7	7	6	2
7	5	5	8	8	4	2	3
4	5	6	5	8	1	7	3
3	3	3	6	1	8	8	3
1	7	3	2	3	6	4	8
1	6	2	2	4	5	8	7

6	4	5	7	7	3	3	5
4	8	4	3	6	7	5	1
3	1	5	7	7	7	6	2
7	5	5	8	8	4	2	3
4	5	6	5	8	1	7	3
3	3	3	6	1	8	8	3
1	7	3	2	3	6	4	8
1	6	2	2	4	5	8	7

ANSWER : _____

World Youth Mathematics Intercity Competition

Team Contest Time limit: 60 minutes 2008/10/28

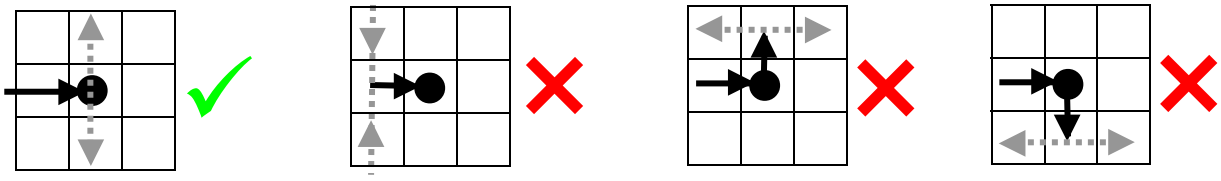
ChiangMai, Thailand

3. On the following 8×8 board, draw a single path going between squares with common sides so that

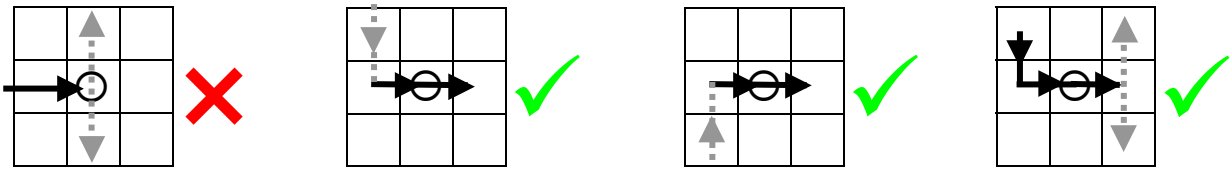
(a) it is closed and not self-intersecting;

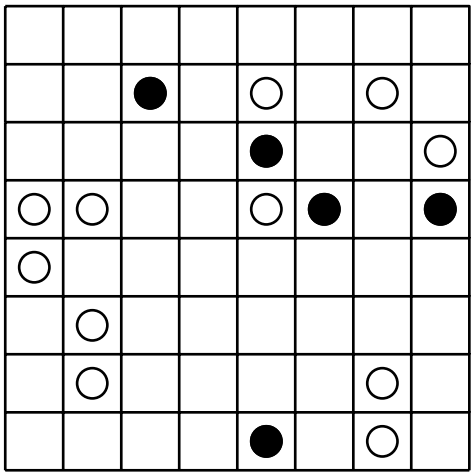
(b) it passes through every square with a circle, though not necessarily every square;

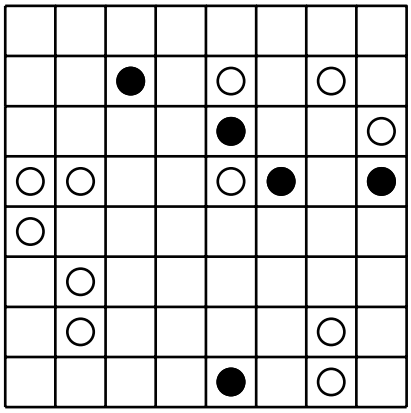
(c) it turns at every square with a black circle, but does not do so on either the square before or the one after;



(d) it does not turn at any square with a white circle, but must do so on either the square before or the one after, or both.







ANSWER : _____

World Youth Mathematics Intercity Competition

Team Contest Time limit: 60 minutes 2008/10/28

Chiang Mai, Thailand

Team: _____ ***Score:*** _____

4. Consider all $a \times b \times c$ boxes where a, b and c are integers such that $1 \le a \le b \le c \le 5$. An $a_1 \times b_1 \times c_1$ box fits inside an $a_2 \times b_2 \times c_2$ box if and only if $a_1 \le a_2$, $b_1 \le b_2$ and $c_1 \le c_2$. Determine the largest number of the boxes under consideration such that none of them fits inside another.

ANSWER : _____

World Youth Mathematics Intercity Competition

Team Contest Time limit: 60 minutes 2008/10/28

Chiang Mai, Thailand

Team: _____ ***Score:*** _____

5. Initially, the numbers 0, 1 and 4 are on the blackboard. Our task is to add more numbers on the blackboard by using the following procedures: In each step, we select two numbers a and b on the blackboard and add the new number $c=ab+a+b$ on the blackboard. What is the smallest number not less than 2008 which can appear on the blackboard after repeating the same procedure for several times?

ANSWER : _____

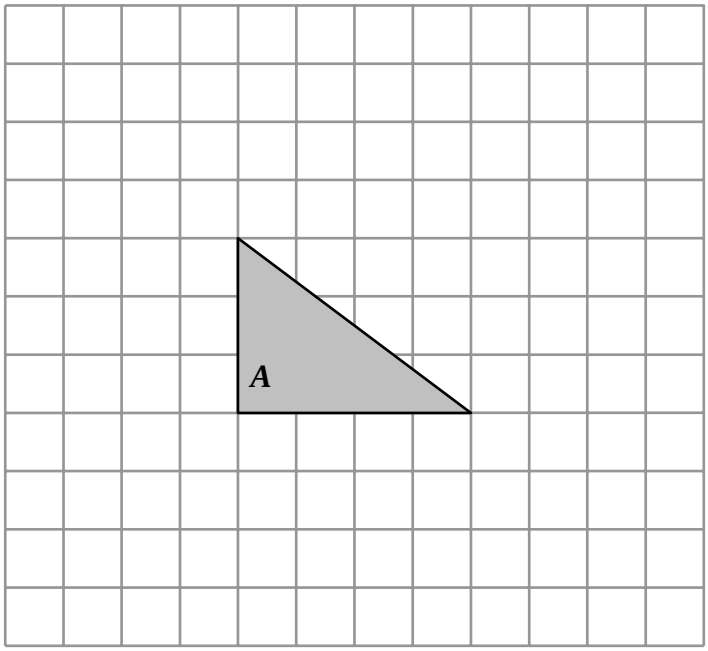
World Youth Mathematics Intercity Competition

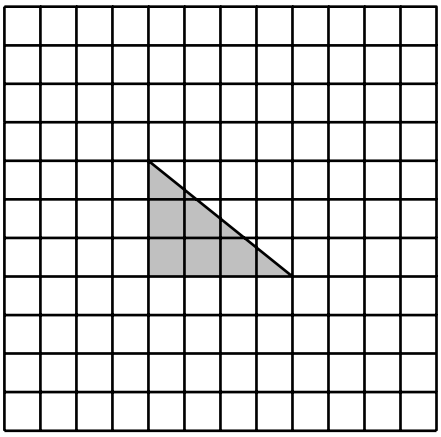
Team Contest Time limit: 60 minutes 2008/10/28

Chiang Mai, Thailand

Team: _____ ***Score:*** _____

6. Given a shaded triangle as below, find all possible ways of extending one of its sides to a new point so that the resulting triangle has two equal sides. Mark the points of extension on the space given below.





ANSWER : _____

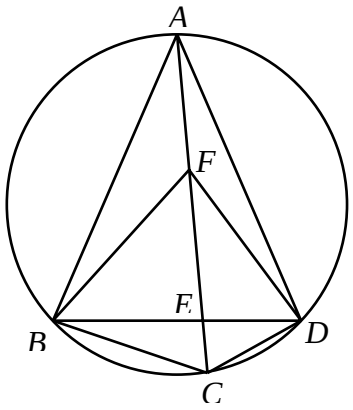
World Youth Mathematics Intercity Competition

Team Contest Time limit: 60 minutes 2008/10/28

Chiang Mai, Thailand

Team: _____ ***Score:*** _____

7. *ABCD* is a quadrilateral inscribed in a circle, with *AB*=*AD*. The diagonals intersect at *E*. *F* is a point on *AC* such that $\angle BFC=\angle BAD$. If $\angle BAD=2\angle DFC$, determine $\frac{BE}{DE}$.



ANSWER : _____

World Youth Mathematics Intercity Competition

Team Contest Time limit: 60 minutes 2008/10/28

Chiang Mai, Thailand

Team: _____ ***Score:*** _____

8. How many five-digit numbers are there that contain the digit 3 at least once?

ANSWER : _____

World Youth Mathematics Intercity Competition

Team Contest Time limit: 60 minutes 2008/10/28

Chiang Mai, Thailand

Team: _____ ***Score:*** _____

9. Among nine identically looking coins, one of them weighs a grams, seven of them b grams each and the last one c grams, where $a<b<c$. We wish to determine whether $a+c<2b$, $a+c=2b$ or $a+c>2b$ using only an unmarked beam balance four times.

ANSWER : _____

World Youth Mathematics Intercity Competition

Team Contest Time limit: 60 minutes 2008/10/28

Chiang Mai, Thailand

Team: _____ ***Score:*** _____

10. Determine the sum of all positive integers n such that

$$1+n+\frac{n(n-1)}{2}+\frac{n(n-1)(n-2)}{6}=2^k$$ for some positive integer k.

ANSWER : _____

International Mathematics Competition 2008
(IMC 2008)

World Youth Mathematics Intercity Competition

Team Contest Time limit: 60 minutes 2008/10/28

Chiang Mai, Thailand

Team: _____ ***Score:*** _____

ANSWER : _____